South West Schools' Federation

Progression in Calculation Document

Revised: October 2016

This document should be read in conjunction with following pages in the Inspire Maths Teacher guides for each year group; 'Bar Model' on page viii, 'Heuristic for Problem Solving' on page ix and 'Making use of Variation' on page x.
This document is intended to give a progression of examples of manipulatives you can use in your teaching. Therefore, you should be using a range of manipulatives to support children understanding and progression in maths.

Addition

Objective and Strategies	Concrete	Pictorial	Abstract

Combining two parts to make a whole: part- whole model	Use cubes to add two numbers together as a group or in a bar. Use cubes to add two numbers together as a group or in a bar. 8 1 8 1	Use pictures to add two numbers together as a group or in a bar. Use pictures to add two numbers together as a group or in a bar.	4 + 3 = 7 5 3 5 3 Use the part-part whole diagram as shown above to move into the abstract. Use the part-part whole diagram as shown above to move into the abstract.

Re-grouping to make 10.	6 + 5 = 11 Use Numicon and straws to show regrouping to make 10. Use Numicon and straws to show regrouping to make 10. Start with the bigger number and use the smaller number to make 10.	Use pictures or a number line. Regroup or partition the smaller number to make 10.	7 + 4= 11 If I am at seven, how many more do I need to make 10. How many more do I add on now?
As an alternate way of adding bigger number from 10. As an alternate way of adding bigger number from 10.	Start with the larger number on the bead string and then count on to the smaller number 1 by 1 to find the answer.	12 + 5 = 17 Start at the larger number on the number line and count on in ones or in one jump to find the answer.	5 + 12 = 17 Place the larger number in your head and count on the smaller number to find your answer.

Adding three single digits	4 + 7 + 6= 17 Put 4 and 6 together to make 10. Add on 7. Add together three groups of objects. Draw a picture to recombine the groups to make 10. Add together three groups of objects. Draw a picture to recombine the groups to make 10.	+ + + + + +	Combine the two numbers that make 10 and then add on the remainder.
Column method. No re-grouping	24 + 15= Add together the ones first then add the tens. Use the Base 10 blocks first before moving onto place value counters.	After practically using the base 10 blocks and place value counters, children can draw the counters to help them to solve additions. T O Use a number line to to show the jumps when adding larger numbers together. 21 + 16 =	

Column method. Regrouping	This is done with Base 10 to help children clearly see that 10 ones equal 1 ten and 10 tens equal 100. Make both numbers on a place value grid. Add up the units and exchange 10 ones for one 10. Add up the rest of the columns, exchanging the 10 counters from one column for the next place value column until every column has been added. As children move on to decimals, money and decimal place value counters can be used to support learning.	Children can draw a pictoral representation of the columns and place value counters to further support their learning and understanding. They can also use the number line method here.	Start by partitioning the numbers. Then add the numers together from the right hand side (O-T-H-TH) H TO As the children move on, introduce decimals with the same number of decimal places and different. Money can be used here.

Subtraction

Objective and Strategies	Concrete	Pictorial	Abstract

Taking away ones	Use physical objects, counters, cubes etc to show how objects can be taken away. 6 – 2 = 4	Cross out drawn objects to show what has been taken away.	18 -3= 15 8 – 2 = 6
Counting back and counting on	Make the larger number in your subtraction. Move the beads along your bead string as you count backwards in ones. 13 – 4 Use counters and move them away from the group as you take them away counting backwards as you go.	Count back on a number line or number track Start at the bigger number and count back the smaller number showing the jumps on the number line. This can progress all the way to counting back using two 2 digit numbers. You can also use the counting on method on a number line.	Put 13 in your head, count back 4. What number are you at? Use your fingers to help.

Part-Part Whole Model	Link to addition- use the part whole model to help explain the inverse between addition and subtraction. If 10 is the whole and 6 is one of the parts. What is the other part? 10 - 6 =	Use a pictorial representation of objects to show the part part whole model.	10 5 10 5 Move to using numbers within the part whole model. 8 – 8 = 0
Find the difference	Compare amounts and objects to find the difference. Use cubes to build towers or make bars to find the difference Use basic bar models with items to find the difference	Count on to find the difference. Draw bars to find the difference between 2 numbers.	Hannah has 23 sandwiches, Helen has 15 sandwiches. Find the difference between the number of sandwiches.
Make 10	14 – 9 = Make 14 on the ten frame. Take away the four first to make 10 and then takeaway one more so you have taken away 5. You are left with the answer of 9.	Start at 13. Take away 3 to reach 10. Then take away the remaining 4 so you have taken away 7 altogether. You have reached your answer.	16 – 8= How many do we take off to reach the next 10? How many do we have left to take off?

Column method without re-grouping	Use Base 10 to make the bigger number then take the smaller number away. Show how you partition numbers to subtract. Again make the larger number first.	Draw the Base 10 or place value counters alongside the written calculation to help to show working.	Write a clear written column subtraction. (Always use the place value names above the calculations). TO

Column method with re-grouping	Use Base 10 to start with before moving on to place value counters. Start with one exchange before moving onto subtractions with 2 exchanges. Make the larger number with the place value counters Start with the ones, can I take away 8 from 4 easily? I need to exchange one of my tens for ten ones. Now I can subtract my ones. Now look at the tens, can I take away 8 tens easily? I need to exchange one hundred for ten tens. Now I can take away eight tens and complete my subtraction Show children how the concrete method links to the written method alongside your working. Cross out the numbers when exchanging and show where we write our new amount.	Draw the counters onto a place value grid and show what you have taken away by crossing the counters out as well as clearly showing the exchanges you make. When confident, children can find their own way to record the exchange/regrouping. Just writing the numbers as shown here shows that the child understands the method and knows when to exchange/regroup.	Children can start their formal written method by partitioning the number into clear place value columns. Moving forward the children use a more compact method. This will lead to an understanding of subtracting any number including decimals.

Multiplication

Objective and Strategies	Concrete	Pictorial	Abstract
Doubling	Use practical activities to show how to double a number.	Draw pictures to show how to double a number.	Partition a number and then double each part before recombining it back together.

Counting in multiples	Count in multiples supported by concrete objects in equal groups.	Use skip counting to count across the multiplies. Use a number line or pictures to continue support in counting in multiples.	Count in multiples of a number aloud. Write sequences with multiples of numbers. 2, 4, 6, 8, 10 5, 10, 15, 20, 25 , 30
Repeated addition	Use different objects to add equal groups. Use different objects to add equal groups.		Write addition sentences to describe objects and pictures. 5 twos = 10 5 groups of 2 = 10

Arrays- showing commutative multiplication	Create arrays using counters/ cubes to show multiplication sentences.	Draw arrays in different rotations to find **commutative** multiplication sentences. Link arrays to area of rectangles.	Use an array to write multiplication sentences and reinforce repeated addition. 3 groups of 5 = 15 3 + 3 + 3 + 3 + 3 = 5x3 = 15
Multiplication models	Create bar models using cubes to show the groups.	The next step will be to draw visual representations of the bar models with the numbers. 5 5 5 5 20 Ensure children understand that a cube represents a plate/packet/basket, etc.	Peter puts 5 bread rolls into each packet. He has 4 packets. How many bread rolls does he put into the 4 packets altogether? 4 x 5 = 20

Division

Objective and Strategies	Concrete	Pictorial	Abstract
Sharing objects into groups	I have 10 cubes, can you share them equally in 2 groups?	Children use pictures or shapes to share quantities. 8 ÷ 2 = 4 8 ÷ 2 = 4	Share 9 buns between three people. 9 ÷ 3 = 3
Division as grouping	Divide quantities into equal groups. Use cubes, counters, objects or place value counters to aid understanding.	Use a number line to show jumps in groups. The number of jumps equals the number of groups.	28 ÷ 7 = 4 Divide 28 into 7 groups. How many are in each group?
Division within arrays	Link division to multiplication by creating an array and thinking about the number sentences that can be created. Eg 15 ÷ 3 = 5 5 x 3 = 15 15 ÷ 5 = 3 3 x 5 = 15	Draw an array and use lines to split the array into groups to make multiplication and division sentences.	Find the inverse of multiplication and division sentences by creating four linking number sentences. 7 x 4 = 28 4 x 7 = 28 28 ÷ 7 = 4 28 ÷ 4 = 7

Division with a remainder	14 ÷ 3 = Divide objects between groups and see how much is left over	Jump forward in equal jumps on a number line then see how many more you need to jump to find a remainder. Draw dots and group them to divide an amount and clearly show a remainder.	Complete written divisions and show the remainder using r.
Division models	Create bar models using cubes to show the groups.	Think of the bar as a whole. Split it into the number of groups you are dividing by and work out how many would be within each group.	Hardeep buys 12 pears. He puts an equal number of pears into 3 boxes. How many pears are there in each box? 12 ÷ 3 = 4 4 x 3 = 12

Fractions

Objective and Strategies	Concrete	Pictorial	Abstract

Making equal parts	Make equal parts out of different paper shapes.	Shade in the parts of the fraction. 1/5 ¼	1/5 is 1 out of the 5 equal parts. 5/5 is a whole. ¼ = one fourth 2/2 = one whole
Part-Part whole	Create models using couloured cubes. Using the fraction of 2/5 to create the model. 2 part red. 5 total parts.	The next stage is to draw the a visual representation of the model. 2 parts are red. 3 parts are orange.	Number of red parts = 2 Number of parts altogether = 5. The fraction of the whole in red is 2/5. The fraction of the whole in yellow is 3/5. 2/5 and 3/5 make 1 whole.

Comparing and ordering	Comapre fractions using cubes. Children can visually see which parts are more.	Children can then draw the bar models to represent which fractions are smaller or greater. Use bar models to show equilavlent fractions.   	Greatest 3/6 2/6 1/6 Smallest Ordering fractions 1/11, 1/10, 1/8, 1/5 Equivalent fractions ½ = 2/4

Adding and subtracting	Create the fractions using cubes. 1/5 + 3/5 = Write the fractions to show the parts.	Using the model, draw a visual bar model for adding and subtracting fractions. 1 fifth + 3 fifths = 4 fifths 6 sevenths – 2 sevenths = 4 sevenths	Ella eats 1/5 of a pizza. Tai eats 3/5 of it, What fractions of the pizza do they eat? 1/5 + 3/5 = 4/5 They eat 4/5 of the pizza altogether. 6/7 – 2/7 = 4/7

Finding fractions of a number	Using arrays to make the fraction. 1/6 of 24 1/5 of 15 You can also use the bar model to represent the fraction.	1/5 of 15 = 5 2/5 of 15 = 10 Draw an array and use lines to split the array into groups using knowledge of multiplication and division. ¼ of 20 ? | | | | Visually draw the bar model to represent the fraction. 5 | 5 | 5 | 5 ¼ of 20 = 4 Once a bar model has been drawn and the missing number calculated, the children will be able to work out different parts of the fraction.	One fifth of fifteen. 1/5 of 15 = 5 2/5 of 15 = 10 3/5 of 15 = 15